Complex Impedance

Complex # $i = j = \sqrt{-1}$

seems strange at first, but simple consequence of algebra

Integers (Counting) ⟹ Addition, ~~subtraction~~ multiplication, powers.

Leads to inverse operations ⟹ subtraction, division, Square root

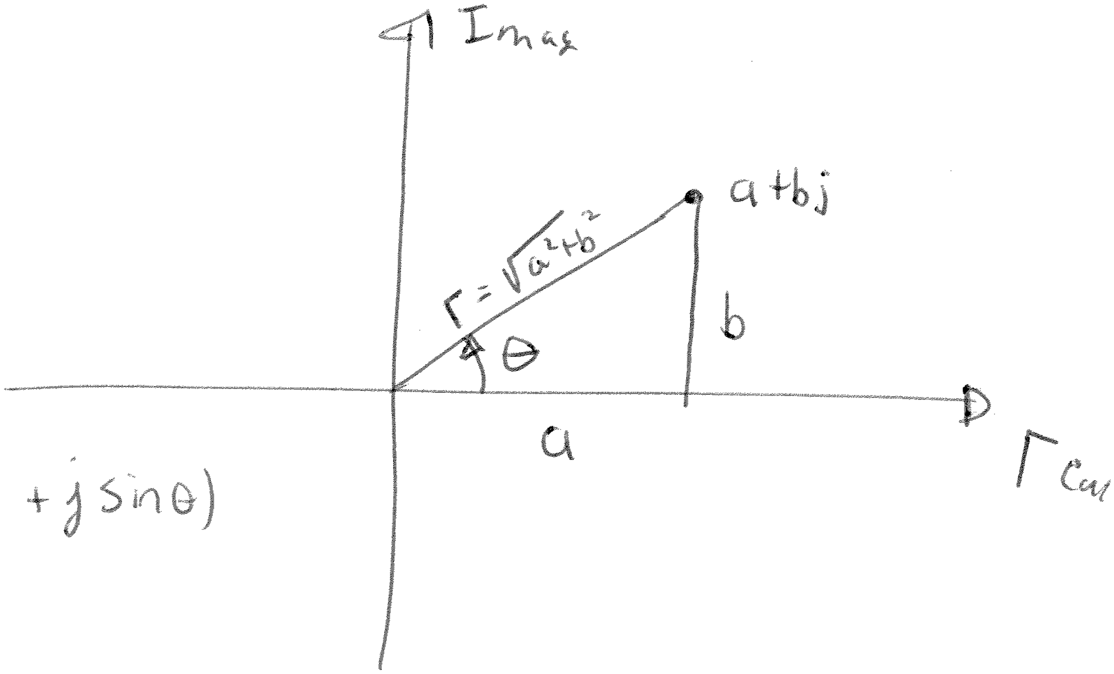
negative #s

Irrational #'s

this set of operations cannot solve every algebraic eqn.

Complex # ⟹ $x^2 = -1$

$e^{j\theta} = \cos\theta + j\sin\theta$ (Proved in linearity)



$r\cos\theta = a$

$r\sin\theta = b$

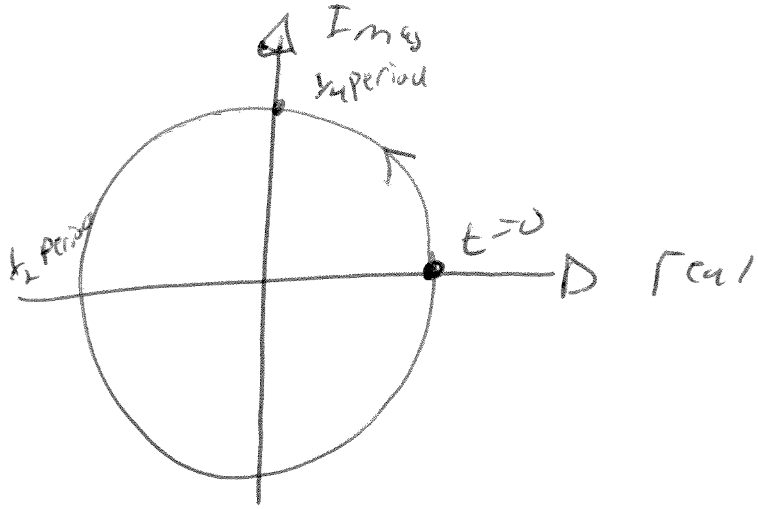
$a+bj = r(\cos\theta + j\sin\theta)$

$= re^{j\theta}$

Since $e^{j\theta} = \cos\theta + j\sin\theta$

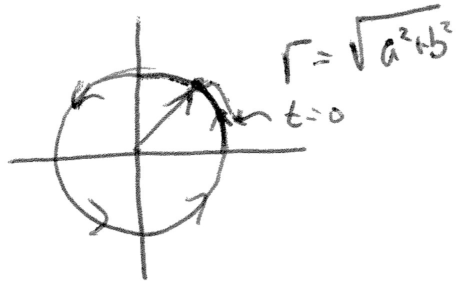
we'll ~~@~~ use $e^{j\omega t}$ to represent sinusoids.

$$V(t) = e^{j\omega t} = \cos\omega t + j\sin\omega t$$



if we add a # out front, a complex amplitude

$$V(t) = \bar{V} e^{j\omega t}$$



Linear Eqns $\Rightarrow$ if we assume complex solns

then real + imag parts are seperable, both must

be satisfied.

Can take real part at the end.

$$V(t) = \bar{V} e^{j\omega t} \Rightarrow V(t) = \text{real}\left(\bar{V} e^{j\omega t}\right)$$

$$\bar{V} = (a + bj)$$

$$V(t) = \text{real}\left((a+bj)\left(\cos\omega t + j\sin\omega t\right) \right)$$

$$= \text{real}\left[\left(a\cos\omega t - b\sin\omega t\right) + j\left(b\cos\omega t + a\sin\omega t\right) \right]$$

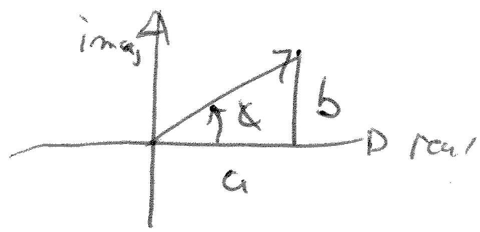
$$V(t) = a\cos\omega t - b\sin\omega t$$

$$V(t) = \sqrt{a^2+b^2}\left[\cos(\omega t)\frac{a}{\sqrt{a^2+b^2}} - \frac{b}{\sqrt{a^2+b^2}}\sin\omega t \right]$$

trig identity

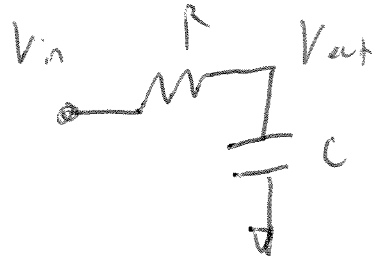
$$\cos(\omega t + \alpha) = \cos\omega t\cos\alpha - \sin\omega t\sin\alpha$$

$$\frac{a}{\sqrt{a^2+b^2}} = \cos\alpha \qquad \frac{b}{\sqrt{a^2+b^2}} = \sin\alpha$$



Phase is somehow built into complex amplitud

Solve some problems and then come back to basics.



sinuso. steady state response

$$V_{in}(t) = \bar{V}_{in}\, e^{j\omega t}$$

$$V_{out}(t) = \bar{V}_{out}\, e^{j\omega t}$$

$$i = \frac{V_{in} - V_{out}}{R} = C\frac{dV_o}{dt}$$

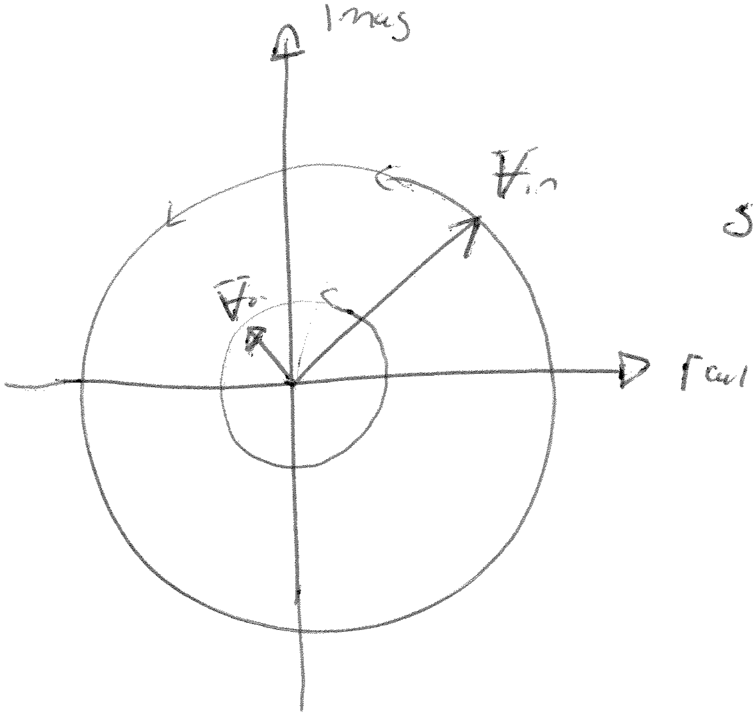
$$RC\frac{dV_o}{dt} = V_{in}(t) - V_{out}(t)$$

$$RCj\omega \bar{V}_o \cancel{e^{j\omega t}} = \bar{V}_{in}\cancel{e^{j\omega t}} - \bar{V}_o\cancel{e^{j\omega t}}$$

$$\bar{V}_{in} = \bar{V}_o\,(1 + RCj\omega)$$

$$\boxed{\frac{\bar{V}_o}{\bar{V}_{in}} = \frac{1}{1 + RCj\omega}}$$

complex #



Same freq. around.

Since starting pt. doesn't matter, only want phase betwee

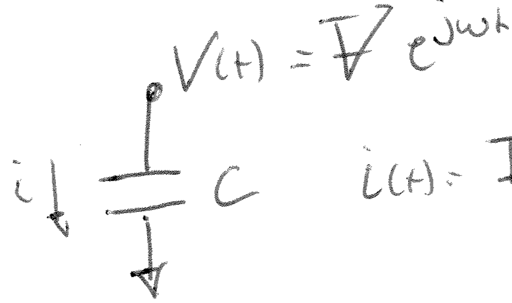
set $\bar{V}_{in} = 1$

then phase is $\operatorname{atan}\left(\frac{b}{a}\right)$

amp. is $\sqrt{a^2+b^2}$

Demo plot in matlab!

A simpler way



$i(t) = I e^{j\omega t}$

$$i(t) = C \frac{dV(t)}{dt}$$

$$\underline{I} = j\omega C \mathbb{V}_0$$

$$\mathbb{V} = I Z \quad \text{when} \quad \boxed{Z = \frac{1}{j\omega C}}$$

Impedance.

for resistor $\boxed{Z = R}$.

Networks of R, C use impedance, and same rules for series and parallel apply!

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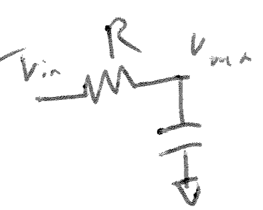
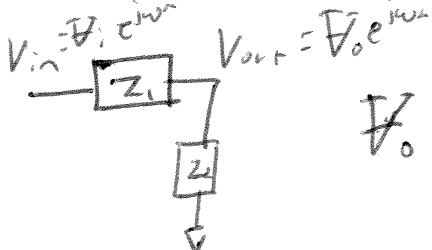
ex

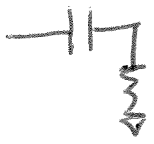


$\equiv$

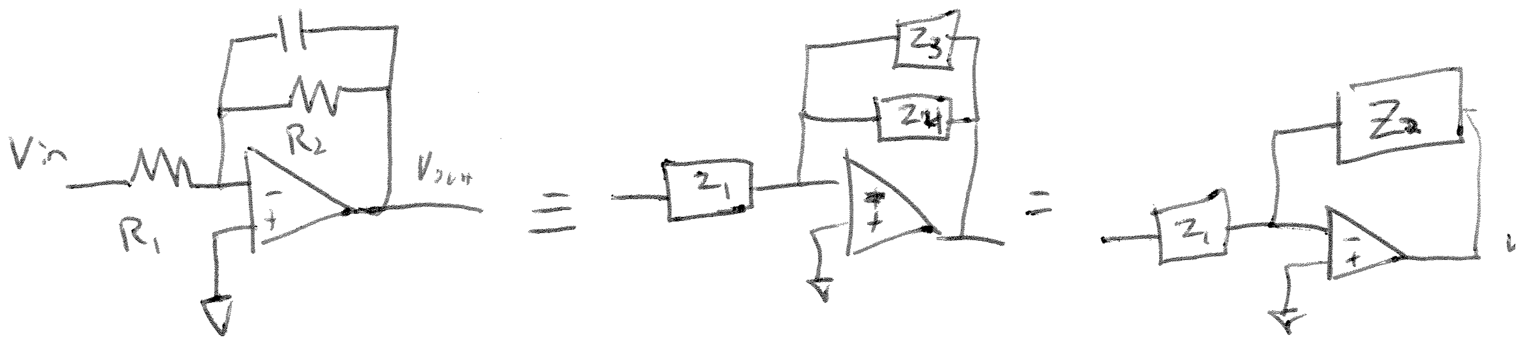


$$\mathbb{V}_0 = \mathbb{V}_{in} \frac{Z_2}{Z_1 + Z_2}$$

$$\frac{\mathbb{V}_0}{\mathbb{V}_{in}} = \frac{1/j\omega C}{\frac{1}{j\omega C} + R} = \frac{1}{1 + j\omega RC}$$



$$= \frac{\mathbb{V}_0}{\mathbb{V}_i} = \frac{R}{\frac{1}{j\omega C} + R} = \frac{j\omega RC}{1 + j\omega RC}$$



$$\frac{V_{in} - 0}{Z_1} = I = \frac{0 - V_{out}}{Z_2}$$

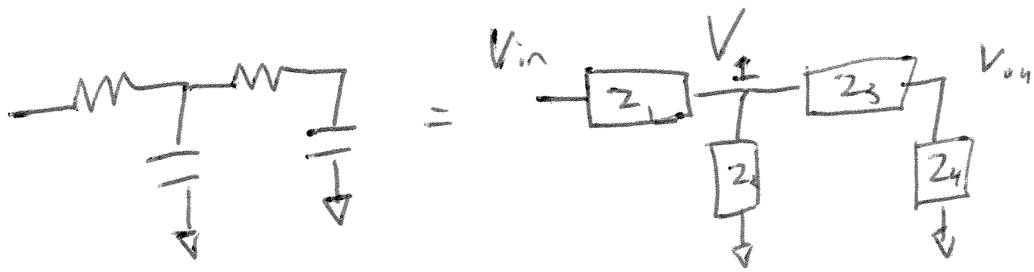
$$\frac{V_o}{V_{in}} = \frac{Z_2}{Z_1}$$

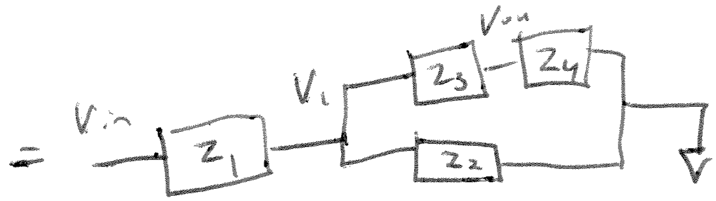
but $$Z_2 = \frac{1}{\frac{1}{Z_3} + \frac{1}{Z_4}} = \frac{1}{\frac{1}{R_2} + j\omega C} = \frac{R_2}{1 + j\omega R_2 C}$$

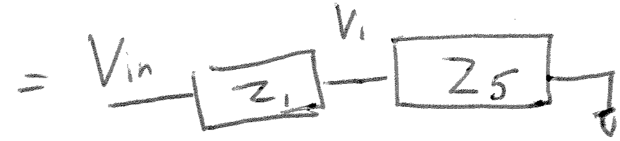
$$\boxed{\frac{V_o}{V_{in}} = \frac{R_2/R_1}{1 + j\omega R_2 C}}$$

gain + filter

if time (Do only if time)

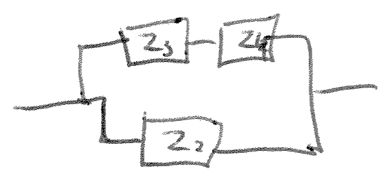






$$V_1 = V_{in} \frac{Z_5}{Z_1 + Z_5}$$

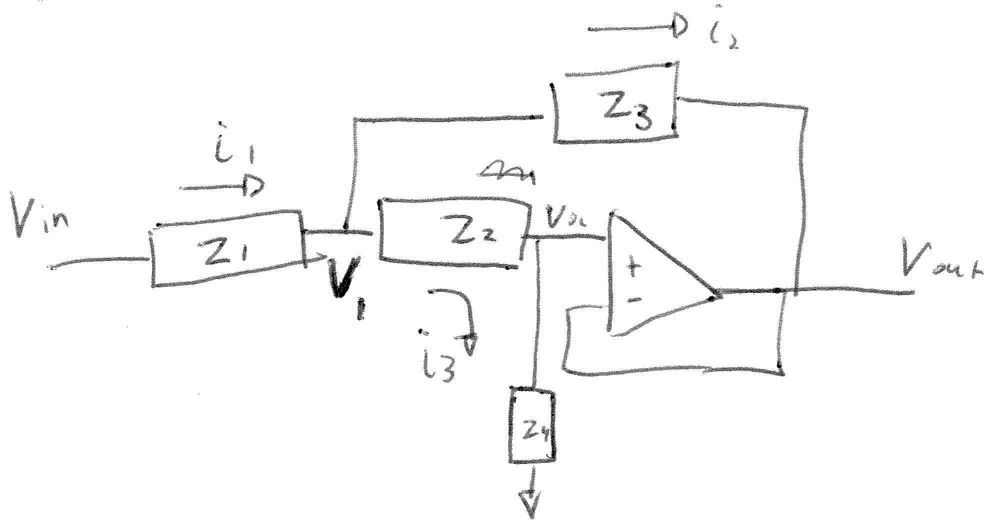
$\boxed{Z_5} =$



$$= \frac{1}{\frac{1}{Z_2} + \frac{1}{Z_3 + Z_4}} = \frac{Z_2}{1 + \frac{Z_2}{Z_3 + 2}}$$

$$V_{out} = V_1 \frac{Z_4}{Z_3 + Z_4} = V_{in} \frac{Z_5}{Z_1 + Z_5} \frac{Z_4}{Z_3 + Z_4}$$

Sallen-Key



$$i_1 = i_2 + i_3$$

Impedance

$$\frac{V_{in} - V_1}{Z_1} = I_1 = I_2 + I_3$$

$$I_2 = \frac{V_1 - V_{out}}{Z_3} \qquad I_3 = \frac{V_1 - V_{out}}{Z_2} = \frac{V_{out}}{Z_4}$$

$$\Downarrow \qquad\qquad \Downarrow$$

$$I_2 = \frac{Z_2}{Z_3 Z_4} V_{out} \qquad V_1 - V_{out} = \frac{Z_2}{Z_4} V_{out}$$

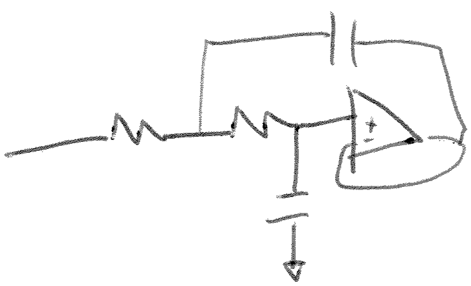
$$V_1 = V_{out}\left(1 + Z_2/Z_4\right)$$

$$\frac{V_{in}}{Z_1} = \frac{V_1}{Z_1} + \frac{Z_2}{Z_3 Z_4} V_{out} + \frac{1}{Z_4} V_{out}$$

$$V_{in} = V_{out}\left[\left(1 + \frac{Z_2}{Z_4}\right) + \frac{Z_1 Z_2}{Z_3 Z_4} + \frac{Z_1}{Z_4}\right]$$

$$= V_{out}\left(\frac{Z_3 Z_4 + Z_2 Z_3 + Z_1 Z_2 + Z_1 Z_3}{Z_3 Z_4}\right)$$

Show examples.



and